

HW SOL 1.5b

November 9, 2017 8:27 PM

Name: _____

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M10 Honours: Section 1.5b Inverse of a Quadratic Functions

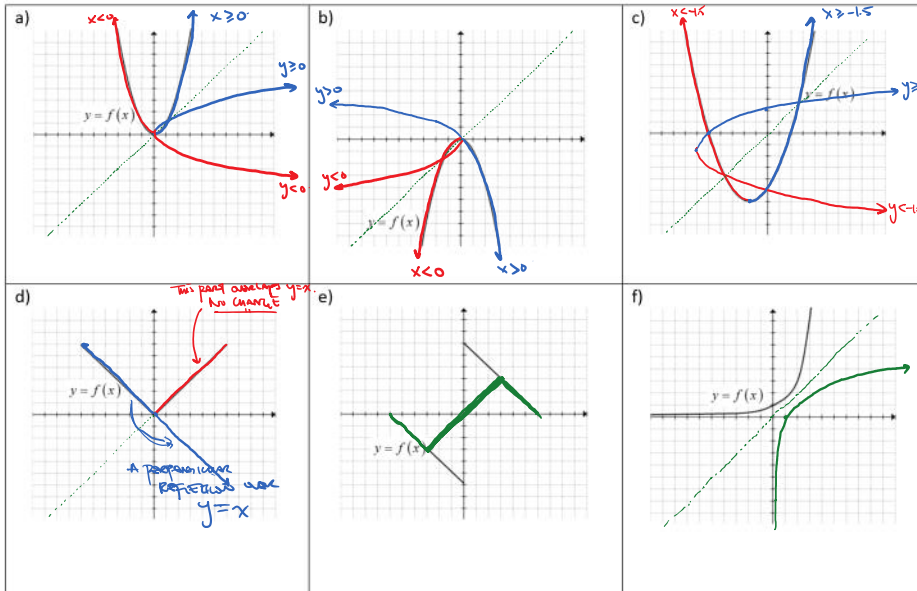
1. Given each equation for $y = f(x)$, find the inverse equation $g(x) = f^{-1}(x)$.

<p>a) $y = 3x - 4$</p> $x = 3y - 4$ $\frac{x+4}{3} = y = f^{-1}(x)$	<p>b) $y = \frac{-8x+11}{2}$</p> $x = \frac{-8y+11}{2}$ $2x = -8y + 11$ $2x - 11 = -8y$ $-\frac{2x-11}{8} = y = f^{-1}(x)$	<p>c) $y = \frac{2-3x}{4+7x}$</p> $x = \frac{2-3y}{4+7y}$ $4x + 7xy = 2 - 3y$ $7xy + 3y = 2 - 4x$ $y(3+7x) = 2-4x$ $y = \frac{2-4x}{3+7x}$
<p>d) $y = \frac{2x-1}{3x+1}$</p> $x = \frac{2y-1}{3y+1}$ $3xy - x = 2y - 1$ $3xy - 2y = x - 1$ $y(3x-2) = x-1$ $y = \frac{x-1}{3x-2} = f^{-1}(x)$	<p>e) $y = -2x^2; x \geq 0$ (RIGHT)</p> $x = -2y^2$ $-\frac{x}{2} = y^2$ $+\sqrt{\frac{-x}{2}} = y = f^{-1}(x)$	<p>f) $y = 3(x-5)^2; x \geq 5$ (RIGHT)</p> $x = 3(y-5)^2$ $\frac{x}{3} = (y-5)^2$ $+\sqrt{\frac{x}{3}} = y - 5$ $5 + \sqrt{\frac{x}{3}} = y = f^{-1}(x)$
<p>g) $y = (x-3)^2 + 1; x \geq 3$ (RIGHT)</p> $x = (y-3)^2 + 1$ $x-1 = (y-3)^2$ $+\sqrt{x-1} = y-3$ $3 + \sqrt{x-1} = y = f^{-1}(x)$	<p>h) $y = -(x+2)^2 - 5; x < -2$ (LEFT)</p> $x = -(y+2)^2 - 5$ $\frac{x+5}{-1} = (y+2)^2$ $-\sqrt{-(x+5)} = y+2$ $-2 - \sqrt{-(x+5)} = y = f^{-1}(x)$	<p>i) $y = -3(x+5)^2 + 6; x < -3$ (LEFT)</p> $x = -3(y+5)^2 + 6$ $\frac{x-6}{-3} = (y+5)^2$ $-\sqrt{\frac{x-6}{-3}} = y+5$ $-5 - \sqrt{\frac{x-6}{-3}} = y = f^{-1}(x)$

<p>j) $y = 2x^2 - 8x + 11; x \geq 2$</p> <p>$y = 2(x^2 - 4x) + 11$ $y = 2(x^2 - 4x + 4) - 8 + 11$ $y = 2(x-2)^2 + 3, x \geq 2$ $x = 2(y-2)^2 + 3$ (RIGHT)</p> <p>$\frac{x-3}{2} = (y-2)^2$ $+\sqrt{\frac{x-3}{2}} = y-2$ $2 + \sqrt{\frac{x-3}{2}} = y$</p>	<p>k) $y = 2x^3 + 6x^2 + 6x + 2$</p> <p>$y = 2(x^3 + 3x^2 + 3x + 1)$ $y = 2(x+1)^3$ $x = 2(y+1)^3$ $\frac{x}{2} = (y+1)^3$ $\sqrt[3]{\frac{x}{2}} = y+1$ $-1 + \sqrt[3]{\frac{x}{2}} = f^{-1}(x)$</p>	<p>l) $y = 5x^3 - 3x^2 + 6x - 12$</p> <p>DONT BOTHER FACTORING: THE INVERSE IS: $x = 5y^3 - 3y^2 + 6y - 12$</p>
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$1 \leftarrow (x+1)^0$
 $1 \ 2 \ 1 \leftarrow (x+1)^2 = x^2 + 2x + 1$
 $1 \ 3 \ 3 \ 1 \leftarrow (x+1)^3 = x^3 + 3x^2 + 3x + 1$
 $1 \ 4 \ 6 \ 4 \ 1$

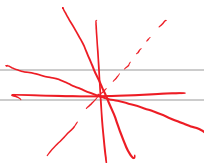
2. Graph $y = f^{-1}(x)$ for each function on the same grid. Restrict the domain if necessary:



3. The following points (3,5), (-3,-7), (-2,8), (7,-10), and (-3,-9) are on the function $y = f(x)$.

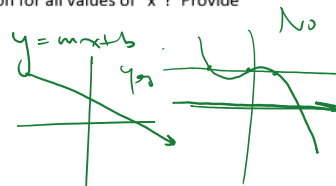
What will the coordinates be on the function: $y = f^{-1}(x)$?
 AN INVERSE FUNCTION WILL SWITCH THE X, Y COORDINATES

$(3,5) \rightarrow (5,3)$ $(7,-10) \rightarrow (-10,7)$
 $(-3,-7) \rightarrow (-7,-3)$ $(-3,-9) \rightarrow (-9,-3)$
 $(-2,8) \rightarrow (8,-2)$



4. Under what conditions will both $y = f(x)$ and $y = f^{-1}(x)$ be a function for all values of "x"? Provide examples of such functions:

IF THE FUNCTION IS ONE TO ONE.
 EACH X GENERATES ONLY ONE Y.
 EACH Y CAN ONLY BE GENERATED BY ONE X.
 i.e.: Graph is continuously INCR. or DECR.



5. Given the function of the parabola, find the equation, domain, and range of the inverse function.

Note: The domain is restricted so that the inverse is also a function.

i) $y = 2(x-3)^2 + 1; x \geq 3$ iv) $y = 0.2x^2 - 2x + 6; x \geq 5$

ii) $y = -3(x+4)^2 - 5; x < -4$ v) $y = \frac{2}{3}x^2 + 8x + 14; x < -6$

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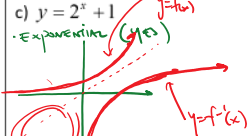
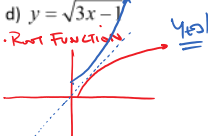
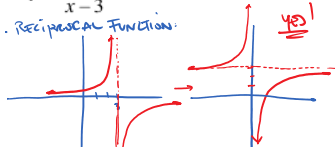
v) $y = \frac{2}{3}x^2 + 8x + 14; x < -6$

6. If $f(3) = -5$ and $f(-5) = 7$, then what is the value of $|f(-5) - f^{-1}(-5)|$?

7. At which points on the graph of $y = f(x)$ will it always intersect the inverse function $y = f^{-1}(x)$?

WHEN THE x COORD & y COORDS ARE EQUAL

8. Given each of the following functions below, please indicate if both $y = f(x)$ and $y = f^{-1}(x)$ are functions for the domain when $x \in \mathbb{R}$. Justify your answer:

<p>a) $y = 3x + 2$. LINEAR FUNCTION (<u>Yes</u>)</p>	<p>b) $y = 3(x-3)^2 + 1$ Quadratic \rightarrow No </p>
<p>c) $y = 2^x + 1$. Exponential (<u>Yes</u>) </p>	<p>d) $y = \sqrt{3x-1}$. Root Function (<u>Yes</u>) </p>
<p>e) $y = \frac{1}{x-3}$. Reciprocal Function (<u>Yes</u>) </p>	<p>f) $y = x^2 - 3x$ Quadratic \rightarrow No!</p>

9. A parabola with equation $y = ax^2 + bx + c$ is reflected about the x-axis. The parabola and its reflection are translated horizontally five units in opposite directions to become graphs of $y = f(x)$ and $y = g(x)$ respectively. Which of the following describes the graph of $y = (f+g)(x)$? i.e.: $y = f(x) + g(x)$

- a) A parabola tangent to the x-axis b) A parabola not tangent to the x-axis
 c) a horizontal line d) A non-horizontal line e) the graph of a cubic function

$f(x) = y = a(x-p)^2 + q \iff g(x) = y = -a(x-p)^2 - q$
 $y = x^2 + 3 \iff y = -x^2 - 3$
 $y = (x-5)^2 + 3 \iff y = -(x+5)^2 - 3$
 $f(x) = y = x^2 - 10x + 25 + 3$
 $g(x) = -(x^2 + 10x + 25) - 3$
 $= -x^2 - 10x - 25 - 3$
 $f(x) + g(x) = -20x$

13. The parabola with equation $y = ax^2 + bx + c$ and vertex (h, k) is reflected about the line $y = k$. This results in the parabola with equation $y = dx^2 + ex + f$. Which of the following equals $a + b + c + d + e + f$?

(A) $2b$ (B) $2c$ (C) $2a + 2b$ (D) $2h$ (E) $2k$

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